Total No. of Questions : **8]** [Total No. of Printed pages : **4**

(1126)

B.A./B.Sc. (General) Ist Semester (0001) Examination

0043

MATHEMATICS

Paper : I

(Plane Geometry)

Time : 3 Hours] **[Maximum Marks : 65**

Note :- Attempt *five* questions in all, selecting at least two questions from each sections.

Section-A

1. (a) Transform $5x^2 - 2xy + 5y^2 - 10y - 7 = 0$ to rectangular axes through (0,1) inclined at an angle $\frac{\pi}{4}$ to the original axes.

   (b) Show that $x^2 + (a\sqrt{3y} - 3)x + (3y^2 - 3\sqrt{3y} - 4) = 0$ represents a pair of straight line. Also find distance between mean. 3,3

2. (a) Prove that the joint equation of straight lines bisecting the angles between lines :

$$ax^2 + 24xy + by^2 = 0 \text{ is } \frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

(b) Find equation of pair of lines joining the origin to the points of intersection of line $y = mx+c$ with the curve Prove that they are perpendicular If $2c^2 = a^2(1 + m^2)$ 3,3

3. (a) Find the locus of mid-points of the chords of the circle $x^2 + y^2 = 16$ which touch the circle $(x - 4)^2 + (y - 3)^2 = 36$

(b) Find the equation of the circle which passes through the origin and cuts orthogonally each of the circles $x^2 + y^2 - 8x + 12 = 0$ and $x^2 + y^2 - 4x - 6y - 3 = 0$ 3,3

4. (a) The point (2,1) is a limited point of a coaxial system of circle of which $x^2 + y^2 - 4y - 3 = 0$ is 9 member. Find the equation of the radical axis and the co-ordinates of the other limiting point.

(b) Find the equation of circle which passes through the point (2 , 0) and touches the straight line $x + 2y - 1 = 0$ at the point (3 , -3). 3,3

Section-B

5. (a) Prove that the locus of the middle points of the normal chords of the parabola $y^2 = 4ax$ is :

$$\frac{y^2}{2a} + \frac{4a^2}{y^2} = x - 2a$$

(b) Prove that in a parabola the chords of contract of tangents at the eight angles passes through focus. 3,3

6. (a) Show that the minimum angle between a pair of conjugate diameter of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$$\tan^{-1}\left(\frac{2ab}{a^2 - b^2}\right)$$

(b) Prove that the locus of the mid-points of the chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ which touch the circle on the joint of the foci of the ellipse as diameter is :

$$\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^2 = a^2 c^2 \left(\frac{x^2}{a^4} + \frac{y^2}{b^4}\right)$$ 3,3

7. (a) Prove that the pole of $px + my = 1$ w.r.t the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ lies on the ellipse

$$\frac{x^2}{9a^2} + \frac{y^2}{9b^2} = 1 \text{ if } a^2p^2 + b^2m^2 = 9$$

(b) If $y = x$ is a diameter of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

and eccentricity of the ellipse is $\frac{1}{\sqrt{3}}$, find the equation of the diameter conjugate to it. 3,3

8. (a) Show that the locus of the mid-points of the chords of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ whose pole lies on the line $x + y - 1 = 0$ is the hyperbola :

$$\frac{x^2}{16} - \frac{y^2}{9} = x + y$$

(b) Find the asymptotes of the hyperbola $xy - x - 2y - 5 = 0$. Also find the equation of the conjugate hyperbola 3,3