

(i) Printed Pages :4]

Roll No.

(ii) Questions :8]

Sub. Code :

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**B.A./B.Sc. (General) 1st Semester
Examination
1127
MATHEMATICS
(Calculus-I)
Paper : II**

Time : 3 Hours]

[Max. Marks : 80

- Note :-** (i) Attempt five questions, selecting at least two questions from each Unit.
(ii) Each question will carry 6 marks.

Unit-I

1. (a) Solve the inequation :

$$\frac{2}{x-2} < \frac{x+2}{x-2} < 2.$$

- (b) State and prove Archimedian property. Using the property prove that the set of natural numbers N is not bounded above.

(3,3)

2. (a) Show that $\lim_{x \rightarrow 0} \sin \frac{1}{x}$ does not exist.

(b) Evaluate :

$$\lim_{x \rightarrow 1/2} \frac{1}{x} \left[\frac{1}{x} \right], \text{ if exists.} \quad (3,3)$$

3. (a) Use intermediate value theorem to show that equation $\sin x - x + 1 = 0$ has a real root.

(b) Evaluate :

$$\lim_{x \rightarrow 0} \frac{x - \sin x}{\tan^3 x} . \quad (3,3)$$

4. (a) Evaluate :

$$\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right) .$$

(b) Discuss the Continuity of

$$f(x) = \left\{ \begin{array}{ll} \frac{|x| + x}{3} , & x \leq 3 \\ \frac{2|x-3|}{x-3} , & x > 3 \end{array} \right\} \text{ over } \mathbb{R}. \quad (3,3)$$

Unit-II

5. (a) Differentiate $y = x^{\sinh x} + x^{\cosh x}$ w.r.t. x .
- (b) Let f be a real valued function defined in $[a, b]$ such that (i) f is continuous in $[a, b]$ (ii) f is differentiable in (a, b) (iii) $f(a) = f(b)$, then there exists at least one $c \in (a, b)$ such that $f'(c) = 0$, (2,4)

6. (a) Prove that $\tanh^{-1} x = \frac{1}{2} \log \left(\frac{x+1}{1-x} \right)$, $-1 < x < 1$,

and then find its derivative.

- (b) Use Cauchy's mean value theorem to evaluate

$$\lim_{x \rightarrow 1} \frac{\frac{\cos \pi x}{2}}{\frac{\log 1}{x}}. \quad (3,3)$$

7. (a) Use mean value theorem to prove :

$$\frac{x}{1+x} < \log(1+x) < x \quad \text{for } x > -1, x \neq 0.$$

- (b) Use Taylor's theorem to express the polynomial $2x^3 + 7x^2 + x - 6$ in powers of $(x - 2)$. (3,3)

8. (a) State and prove Leibnitz's Theorem.

- (b) If $y = \frac{\log x}{x}$, prove that

$$y = \frac{(-1)^n \ln}{x^{n+1}} \left[\log x - 1 \frac{-1}{2} \frac{-1}{3} \dots \dots \frac{-1}{n} \right] \quad (3,3)$$