(ii) Questions :8]

## B.A./B.Sc (General) 4th Semester 1046 <br> MATHEMATICS <br> Paper : III-Dynamics

Time: 3 Hours]
[Max. Marks: $\mathbf{3 0}$

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Note :- Attempt five questions, selecting at least two questions from each Unit. Each question will carry 6 marks.

## UNIT-I

1. (a) Abusis beginning to move with an acceleration $1 \mathrm{~m} / \mathrm{Sec}$. A man who is 40 m behind the bus starts running at $9 \mathrm{~m} / \mathrm{sec}$ to catch the bus. After how many seconds will the man be able to catch the bus ? Explain the double answer. If the man is $40_{1 / 2} \mathrm{~m}$ behind, will he be able to catch the bus?
(b) From the top of a tower 360 m high, a ball is projected downwards with an initial velocity of $10 \mathrm{~m} / \mathrm{sec}$. At the same instant another ball is projected upwards from the bottom with a velocity of $150 \mathrm{~m} / \mathrm{sec}$. Find the distance from the bottom of the tower of the point at which they pass each other. Also, prove that the sum of the speeds of the two ball at any instant is the same.
II. (a) Two particles of masses $m_{1}$ and $m_{2}$ are connected by inextensible string $\mathrm{m}_{2}$ is placed ona smooth horizontal table. The string passes over a light smooth pulley at the edge of the table and $m$ is hanging freely. Find the tension in the string and pressure on the pulley.
(b) A balloon on mass $m$ is rising with an acceleration $f$. Prove that the fraction of weight of balloon that must be detached in order to double its acceleration is $\frac{\mathrm{mg}}{2 f+g}$ assuming that up thrust of air remains same.
III. (a) The acceleration of a particle varies directly as time which has elapsed since its velocity was $10 \mathrm{~m} / \mathrm{sec}$. If it travels 168 min 6 seconds, following this instant, find how far will it travel in next 4 seconds.
(b) A particle starts from rest under the action of its weight and a resistance equal to ' $k v$ ' per unit of mass, where $k$ is a constant and $v$ is velocity. Prove that :
(i) $\mathrm{v}=\frac{\mathrm{g}}{\mathrm{k}}\left(1-\mathrm{e}^{-\mathrm{kt}}\right)$
(ii) $s=\frac{g}{k}\left(k t+e^{-k t}-1\right)$
(iii) $k s+v \frac{g}{k} \log \left|1-\frac{k v}{g}\right|=0$
where $s$ is the distance travelled and $t$ is the time in seconds.
IV. (a) A particle moving with S.H.M. of period 12 seconds travels 10 cm from position of rest in 2 seconds. Find amplitude. maximum velocity and velocity at the end of 2 seconds.
(b) Anelastic string. to the middle of which a particle of mass $m$ is attached, is stretched to twice its natural length and placed on a smooth horizontal table and its ends are then fixed. If the particle is displaced in the direction of string, then find period of oscillation.

## UNIT-II

V. (a) Find expressions for acceleration along the tangent and normal of a particle moving along a plane curve.
(b) A shot is fired horizontally from the top of a tower with a velocity of $200 \mathrm{~m} / \mathrm{sec}$ which hits the ground in 2 sec . What is the height of the tower ? Also, find the distance from the foot of the tower where the shot strikes the ground.
VI. (a) A particle moves down along the outside of a smooth vertical circle of radius r. If it starts from rest at depth
$\frac{r}{2}$ below the highest point, prove that it leaves the circle
at distance $\frac{r}{3}$ above the centre.
(b) A train moving at a rate of $50 \mathrm{~km} / \mathrm{hr}$ is stuck by stone moving with velocity of $40 \mathrm{~km} / \mathrm{hr}$ making an angle of $60^{\circ}$ with direction of train. Find the velocity with whichthe stone appears to a passenger in the train to strike the train.
VII. (a) Prove that work done in stretching an elastic string is equal to the product of the extension and mean of initial and final tensions.
(b) If a particle moves under the action of conservative system of forces, then prove that the sum of its kinetic and potential energies remains constant throughout the motion.
VII. (a) An elastic ball is dropped from a height $h$ above the groud and at the same time a second equal ball is projected vertically upwards to meet the former. Show that the balls come to rest immediately after the impact if the second ball be projected with a velocity which would just carry it to a height $h$.
(b) A mass matter falling freely through distance a begins to raise a a mass $M(M>m)$ connected with it by means of an inextensible string over a smooth pulley. Show that $M$ will return to its original position after time $\frac{2 m}{M-m} \sqrt{\frac{2 a}{g}}$.
Also find the impulse of the jerk and the distance through which M rises.

