[Total No. of (i) Printed Pages 4 (ii) Questions 8]

Sub Code : 0541(1048) **Exam Code :** 0006

Exam : B.A./B.Sc. (General), 6th Semester

Subject : Mathematics

Paper : Paper-I : Analysis-II

Time : 3 Hours ***Maximum Marks : 30***

Note : Attempt five questions in all selecting two questions from each section. All questions carry equal marks.

1. (a) $\iint_{A} x\,dx\,dy$, where A is the region bounded by the parabolas circles :

$$y^2 = 4ax \text{ and } x^2 = 4ay.$$

(b) $\iint \sqrt{a^2 - x^2 - y^2}\,dx\,dy$ over the region bounded by the semi circle $x^2 + y^2 = ax$ lying in the first quadrant.

2. (a) Find the volume of the tetrahedron bounded by the planes: -

$$x = 0, y = 0, z = 0 \text{ and } \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

(b) $\iiint \frac{dx\ dy\ dz}{\sqrt{1 - x^2 - y^2 - z^2}}$ over the positive octant of the sphere $x^2 + y^2 + z^2 = 1$

3. (a) If $\vec{F} = y\hat{i} + (x - 2xz)\hat{J} - xy\hat{k}$, then evaluate

$\iint_S (\nabla \times \vec{F}).\hat{n}\ dS$, where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the XY plane.

(b) If $\vec{F} = (2x^2 + y^2)\hat{i} + (3y - 4x)\hat{J}$, evaluate

$\int_C \vec{F}.\ dr$ around the triangle ABC whose vertices are A(0,0), B(2,0) and C(2,1).

4. (a) Apply Green's theorem in plane to evaluate

$\oint_C [(2x^2 - y^2)\ dx + (x^2 + y^2)dy]$, where C is the boundary of the surface enclosed by the x axis and the semi circle $y = \sqrt{1 - x^2}$.

(b) Verify Stokes Theorem for $\vec{F} - (2x - y)\hat{i} - yz^2\hat{J} - y^2z\hat{K}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

5. (a) Show that the Sequence $\{f_n(x)\}$ defind by $f_n(x) = nxe^{-nx^2}$, converges point wise but not uniformly in $[0, \infty]$

(b) Use M_n — Test to show that the sequence

$\{f_n(x)\}$, where $f_n(x) = \dfrac{nx}{1+n^2x^2}$ does not

converge uniformly on [0,1].

6. (a) Show that the series

$$\frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8} + \ldots\ldots\ldots\ldots\ldots\ldots$$

converges uniformly for $-1 < x < 1$.

(b) Show that the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2 + n^4 x^2}$

is uniformly convergent for all x and it can she differentiated term by term.

7. (a) Find the radii of convergence of the following power series :

(i) $\displaystyle\sum \left(1 + \frac{1}{n}\right)^{n^2} x^n$

(ii) $\sum_{n=0}^{\infty} \frac{(x-2)^{4n}}{4^n}$

(b) Show that $\int_0^1 \frac{\tan^{-1} x}{x} \, dx = \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)^2}$

8. (a) Expand $f(x) = |\cos x|$ as Fourier series in $-\pi < x < \pi$.

(b) Show that for $-\pi \leq x \leq \pi$.

$$x = \frac{\pi^2}{3} - 4\left(\frac{\cos x}{1^2} - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} - \dots\dots\dots\dots\right)$$

Hence evaluate $\sum_{n=0}^{\infty} \frac{1}{n^2}$