Roll No. .... **Total Pages : 4**

1205/MH

AS-2058

ALGEBRA - I

Paper-IV

(Semester-II)

Time Allowed : 3 Hours] **[Maximum Marks : 40**

Note :- The candidates are required to attempt two questions each from Sections A and B carrying 6 marks each and the entire Section C consisting of 8 short answer type questions carrying 2 marks each. .

SECTION-A

I. (a) If A is a Skew-symmetric matrix and (I + A) is non-singular, then show that :

$$B = (I - A)(I + A)^{-1} \text{ is orthogonal.}$$

(b) Use Gauss Jordan method to find the Inverse of

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}.$$ 3,3

II. (a) Prove that the characteristic root of a unitary matrix are of unit modulus.

(b) Find the characteristic equation and eigenvalues of the matrix $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$. What is the spectrum of the matrix ? Also verify that the matrix satisfies its characteristic equation. 3,3

III. (a) Find the values of λ and p so that the system of equations $2x - 3y + 5z = 12$, $3x + y + \lambda z = \mu$ and $x - 7y + 8z = 17$ has (i) Unique solution, (ii) No solution, (iii) Infinite number of solutions.

β

(b) Is the matrix $\begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix}$ diagonalisable ? Justify. 3,3

IV. (a) If A and P be the square matrices of the Same type and P be invertible then show that the matrices A and $P^{-1}AP$ have the same eigenvalues.

(b) If $N = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$, find the matrix $(I - N)(I + N)^{-1}$.
Hence show that matrix obtained is unitary. 3,3

SECTION-B

V. (a) Express $p = \dfrac{(\sqrt{3} - 1) + i(\sqrt{3} + 1)}{2\sqrt{2}}$ in the form of

$r(\cos\theta + i\sin\theta)$ and all values of $p^{1/4}$.

(b) Explain Cardan's method of solving a cubic :

$$a_0x^3 + 3a_1x^2 + 3b_2x + b_3 = 0.$$

3,3

VI. (a) If α, β, γ are roots of $2x^3 - 6x^2 + 3x + k = 0$ such that $\alpha = 2(\beta + \gamma)$. Find k and solve the equation.

(b) If α, β, γ are roots of $x^3 + 3x + 2 = 0$ form an equation whose roots are $(\alpha - \beta)^2$, $(\beta - \gamma)^2$ and $(\gamma - \alpha)^2$. Hence show that the equation has a pair of imaginary roots. 3,3

VII. (a) Sum to n terms the series :

$$\frac{1}{\cos\theta + \cos3\theta} + \frac{1}{\cos\theta + \cos5\theta} + \frac{1}{\cos\theta + \cos7\theta} + \ldots\ldots\ldots\ldots$$

(b) Use Ferrari's method to solve :

$$x^4 - 5x^3 + 3x^2 + 2x + 8 = 0.$$

3,3

VIII. (a) Find 'a' and solve the equation :

$$40x^4 + ax^3 - 21x^2 - 2x + 1 = 0$$

given that roots are in H.P.

(b) What is Descarte's rule of signs. Show that the equation $2x^7 + 3x^4 + 3x + k = 0$ has at least four imaginary roots for all values of k. 3,3

SECTION-C

IX. (a) If α is an eigen value of a non-singular matrix A then prove that $\frac{|A|}{\alpha}$ is an eigen value of Adj A.

(b) Do the matrices A, B, C have the same row space :

$$A = \begin{bmatrix} 1 & -1 & -2 \\ 3 & -2 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 & 5 \\ 2 & 2 & 13 \end{bmatrix} \qquad \text{and}$$

$$C = \begin{bmatrix} 1 & -1 & -1 \\ 4 & -3 & -1 \\ 3 & -1 & 3 \end{bmatrix}$$

(c) Under what conditions $A = \begin{bmatrix} \alpha + i\beta & -\gamma + i\delta \\ \gamma + i\delta & \alpha - i\beta \end{bmatrix}$ is unitary.

(d) Define similar matrices and show that such matrices have same characteristic polynomial and hence same eigen values.

(e) Find the relation between hyperbolic and circular functions.

(f) Show that the equation $x^{11} - x^6 + 4x^5 - 5 = 0$ has at least 8 non-real roots. Can a real root of equation be negative ?

(g) Form an equation whose roots are square of the roots of $x^3 - 3x - 2 = 0$.

(h) Under what conditions roots of :

$$x^3 + 3ax^2 + 3bx + c = 0$$

are in A.P. 8x2