

**1206/MH**

**AS-2058**

**PARTIAL DIFFERENTIAL EQUATION-V**

(Semester-II)

**Time Allowed : 3 Hours]**

**[Maximum Marks : 40**

**Note :-** The candidates are required to attempt two questions each from Sections A and B carrying 6 marks each and the entire Section C. consisting of 8 short answer type questions carrying 2 marks each.

**SECTION-A**

1. (a) Solve :

$$xs + q - xp - z = (1 - y)(1 + \log x).$$

(b) Find the surface passing through the parabolas  $z = 0, y^2 = 4ax$  and  $z = 1, y^2 = -4ax$  and satisfying the equation  $xr + 2p = 0$ .

3,3

2. (a) Find a complete integral of  $p^2 + q^2 - 2px - 2qy + 1 = 0$  using Charpit's method. .
- (b) Find the surface which intersects the surfaces of the system  $z(x + y) = c(3z + 1)$  cut orthogonally and which passes through the circle  $x^2 + y^2 = 1, z = 1$ . 3,3
3. (a) Solve  $(x^2 - y^2 - z^2)p + 2xyq = 2xz$  for general solution.
- (b) Find the equation of integral surface of partial differential equation  $x^2p + y^2q + z^2 = 0$  which passes through the hyperbola  $z = 1, xy = x + y$ . 3,3
4. Reduce the equation  $x^2(y - 1)r - x(y^2 - 1)s + y(y - 1)t + xsp - q = 0$  to canonical form and hence solve it. 6

### SECTION-B

5. (a) Solve partial differential equation :

$$(D^2 + 2DD' + D'^2)z = 2\cos y - x \sin y.$$

(b) Solve partial differential equation :

$$(D^2 - DD' - 2D'^2 + 2D + 2D')z = e^{(x+2y)} + xy. \quad 3,3$$

6. Solve the partial differential equation :

$$(D^2 + DD' - 6D'^2)z = x^2 \sin(x + y). \quad 6$$

7. The temperature at one end of a bar 100 cm long with insulated sides is kept at  $0^\circ\text{C}$  and other end at  $100^\circ\text{C}$  until steady state conditions prevail. The two ends are then suddenly insulated and kept so. Find the temperature distribution. 6

8. A tightly stretched string with fixed ends  $x = 0$  and  $x = l$  is initially in a position given by

$$y = \sin^3 \frac{\pi x}{l}. \text{ Find displacement if it is released from}$$

this position. 6

### SECTION-C

9. Attempt all questions :

1. What are the conditions on partial differential equation to classify Hyperbolic, elliptic or parabolic?

2. Find the complete integral of :

$$(p - q)(z - px - qy) = 1.$$

3. Find the particular integral of :

$$(D^2 - DD' - 3D'^2)z = 10 \frac{e^x}{e^y}.$$

4. Solve partial differential equation :

$$(D^2 - D'^2 + D + 3D' - 2)z = 0.$$

5. Eliminate  $a$  and  $b$  from  $z = axe^y + \frac{1}{2} a^2 e^{2y} + b$

to form partial differential equation.

6. Show that the equation  $xp = yq$  and  $z(xp + yq) = 2xy$  are compatible.

7. Obtain the general solution of Laplace equation.

8. Find the deflection of a vibrating string of unit length having fixed ends with initial velocity zero and initial-deflection is  $f(x) = x - x^3$ .

8x2=16