Roll No. .... **Total Pages : 3**

8696/MH

CS-2057
ALGEBRA-II
Paper-III
(Semester-VI)

Time Allowed : 3 Hours] **[Maximum Marks : 36**

Note :- The candidates are required to attempt two questions each from Sections A and B carrying marks 5.5 each and the entire Section C consisting of 10 questions carrying 1.4 marks each.

SECTION-A

1. (a) Show that any plane passing through (0, 0, 0) is a subspace of R^3. 3

   (b) Examine whether (1, -3, 5) belongs to.the linear space generated by S, where S = {(1, 2, 1), (1, 1, -1), (4, 5, -2)} or not ? 2.5

2. (a) Let V be a vector space of functions $f : R \rightarrow R$. Then show that all f where f (-2) = 0 is a subspace or not of V ? Justify your answer. 3

   (b) State and prove Replacement theorem. 2.5

3. (a) Let V be vector space of 2x2 matrices of R and W be asset of all 2x2 diagonal matrices over R. Show that W is a subspace of V and find basis of V/W. 2.5

(b) If V and W are fine dimensional subspaces of a fine dimensional vector space U(F). Prove that V + W is also finite dimensional and
$\dim(V + W) = \dim V + \dim W - \dim V \cap W$. 3

4. (a) Extend {(-1, 2, 5)} to two different basis of R^3. 2.5

(b) Find basis and dimension of the solution space W of the following system of equations :

$$x + 2y - 4z + 3t - s = 0,$$
$$x + 2y - 2z + 2t + s = 0,$$
$$2x + dy - 2z + 3t + 4s = 0.$$ 3

SECTION-B

5. (a) Find linear transformation $T : R^2 \to R^2$ such that $T(2, 3) = (1, 2)$ and $T(3, 2) = (2, 3)$. 2.5

(b) Prove that non-zero eigen vectors corresponding to distinct eigen values of a linear operator are linearly independent. 3

6. (a) Find range, rank, null space and nullity for $T(x, y) = (x + y, x - y, y)$ transformation on a vector space R^2. 3

(b) Find a linear map $T : R^4 \to R^3$ whose null space is generated by (1, 2, 3, 4) and (0, 1, 1, 1). 2.5

7. Find characteristic and minimal polynomial of a matrix $\begin{bmatrix} 6 & -3 & -2 \\ 4 & -1 & -2 \\ 10 & -5 & -3 \end{bmatrix}$, using idea of minimal polynomial, find inverse of matrix. 5.5

8. (a) If the matrix of the linear operator T on R^3 relative to basis : $B = \{(e_1, e_2, e_3)\} = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ is $\begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Find the matrix of T relative to basis $B_1 = \{(0, -1, 1), (1, -1, -1), (1, -1, 0)\}$. 2.5

(b) Prove that two finite dimensional vector space V(F) and W(F) over the same field F are isomorphic iff they have same dimensions. 3

SECTION-C

9. (a) Show that all polynomials over R with constant term 2 doesn't form vector space.

(b) If S 1s a subspace of V then show that L(S) = S.

(c) Find value of K so that the vector (1, -2, k) becomes linear combination of the vectors (3, 0, -2) and (2, -1, -5).

(d) If S is a subset of V, then prove that $S^{\perp} = \text{span } (S)^{\perp}$.

(e) State and prove Pythagoras theorem for orthogonal set of vectors.

(f) Prove that minimal polynomial of a matrix exist uniquely.

(g) Define singular transformation and Nullity of transformation.

(h) Show that set containing vector 0 is always linear dependent.

(i) The minimal polynomial of an operator T divides every polynomial which has T as a Zero.

(j) Prove that if o is an eigen value ofa linear operator T on V (F) iff α is a root of the miniml polynomial of T.

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