## PC-1246/MH

## CS-2058

## ALGEBRA-II

## Paper-III

(Semester-VI)

Time Allowed : 3 Hours]
[Maximum Marks : 36

Note :- Attempt two questions each from Section $A$ and $B$ carrying 5.5 marks each, and the entire Section-C consisting of 10 questions carrying 1.4 marks each.

## SECTION-A

I. (a) Prove that there exists a basis for finite dimensional vector space.
(b) Examine whether $(1,-3,5)$ belongs to the linear space generated by $S$, where $S=\{(1,2,1),(1,1,-1)$, $(4,5,-2)$ ) or not ?
II. (a) Let V be a vector space of functions $\mathrm{f}: \mathrm{R} \rightarrow \mathrm{R}$. Then show that all $f$ where $f(-2)=0$ is a subspace or not of V? Justify your answer.
(b) State and prove Extension theorem.
III. (a) Let $M$ and $N$ be sub-space of $R^{4}$ defined as $M=\{(a, b, c, d): a+c+d=0), N=\{(a, b, c, d):$ $a=b, d=2 c\}$. Find the dimension and basis of $M, N$ and $M \cap N$.
(b) If V and W are finite dimensional subspaces of a finite dimensional vector space $U(F)$, prove that $V+W$ is also finite dimensional and $\operatorname{dim}(\mathrm{V}+\mathrm{W})=\operatorname{dim} \mathrm{V}+\operatorname{dim} \mathrm{W}-\operatorname{dim} \mathrm{V} \cap \mathrm{W}$.
IV. (a) Extend $\{(-1,2,5)\}$ to two different basis of $R^{3}$.
(b) Find the basis and dimension of the solution space W of the following system of equations:

$$
\begin{align*}
& x+y+z=0 \\
& x+2 y+3 z=0 \\
& x+3 y+4 z=0 \tag{3}
\end{align*}
$$

## SECTION-B

V. (a) Find linear transformation $T: R^{2} \rightarrow R^{2}$ such that $T(2,3)=(1,2)$ and $T(1,2)=(2,3)$.
(b) Find the characteristic and minimal polynomial of a matrix

$$
\left[\begin{array}{lll}
6 & -3 & -2  \tag{3}\\
4 & -1 & -2 \\
10 & -5 & -3
\end{array}\right]
$$

VI. (a) Find range, rank, null space and nullity for $T(x, y)=(x+y, x-y, y)$ transformation on a vector space $R^{2}$.
(b) Find a linear map $T: R^{4} \rightarrow R^{3}$ whose null space is generated by $(1,2,3,4)$ and $(0,1,1,1)$.
VII. Let $T: R^{4} \rightarrow R^{3}$ be a linear transformation defined by $\mathrm{T}\left(\mathrm{x}_{1}, \mathrm{x}_{2}, \mathrm{x}_{3}, \mathrm{x}_{4}\right)=\left(\mathrm{x}_{1}-\mathrm{x}_{2}+\mathrm{x}_{3}+\mathrm{x}_{4}, \mathrm{x}_{1}+2 \mathrm{x}_{3}-\mathrm{x}_{4}, \mathrm{x}_{1}+\mathrm{x}_{2}\right.$ $+3 x_{3}-3 x_{4}$ ) for $x_{1}, x_{2}, x_{3}, x_{4} \in R$. Find the basis and dimension of (i) Range of $T$ (ii) Null space of T. Also verify $\operatorname{Rank}(T)+\operatorname{nullity}(T)=\operatorname{dim}\left(R^{4}\right)$.
VIII. (a) If the matrix of the linear operator $T$ on $R^{3}$ relative to basis $B=\left\{\left(e_{1}, e_{2}, e_{3}\right)\right\}=\{(1,0,0),(0,1,0),(0,0,1)\}$
is $\left[\begin{array}{rrr}0 & -1 & -1 \\ -1 & 0 & 1 \\ 1 & 1 & 0\end{array}\right]$, find the matrix of $T$ relative to
basis $B_{1}=\{(0,-1,1),(1,-1,-1),(1,-1,0)\}$.
(b) Show that the linear transformation $T: R^{3} \rightarrow R^{3}$ defined by $T\left(e_{1}\right)=e_{1}-e_{2},>T\left(e_{2}\right)=2 e_{2}+e_{3}, T\left(e_{3}\right)=e_{1}$ $+e_{3}+e_{2}$, where $\left\{e_{1}, e_{2}, e_{3}\right\}$ is standard basis of $R^{3}$, is neither one-one nor onto.

## SECTION-C

IX. Attempt all the following :
(a) Show that all polynomials over R with no constant term form: a vector space.
(b) Show the union of two subspaces may not be a subspace.
(c) Find the value of k so thet the vector $(1,-2, k)$ becomes linear combination of the vectors ( $3,0,-2$ ) and $(2,-1,-5)$.
(d) Find symmetric orthogonal matrix whose first row is $\left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$.
(e) State and prove Pythagoras theorem for orthogonal set of vectors.
(f) If T is a linear operator on V such that $\mathrm{T}^{2}-\mathrm{T}+\mathrm{I}=0$. prove T is invertible. Define Singular transformation and Nullity of transformation.
(g) Show that the set containing vector 0 is always linear dependent.
(h) Define Characteristic and Minimal polynomial.
(i) Define Eigen values and Eigen vectors.
(j) Prove that minimal polynomial of a matrix exists uniquely.
(10×1.4=14)

