Roll No. ..... **Total Pages : 3**

PC-1248/MH

CS-2058
MATHEMATICAL METHODS-II
Opt.(iii)
(Semester-VI)

Time Allowed : 3 Hours] **[Maximum Marks : 36**

Note :- Attempt five qustions in all selecting two questions from each Section-A and B. Section-C is compulsory.

SECTION-A

(2x5.5=11)

I. (a) Find the Fourier sine and cosine transform of $f(t) = t^{n-1}$.

(b) State and prove convolution theorem for Fourier transforms.

II. (a) Using Parseval's identity, prove that

$$\int_0^\infty \frac{dx}{(a^2 + x^2)(b^2 + x^2)} = \frac{\pi}{2ab\,(a + b)} .$$

(b) Using Fourier integral formula, solve for

$$f(x) = \begin{cases} 2, & -1 \le x \le 1 \\ 0 & x < -1 \text{ or } x > 1 \end{cases}$$

and evaluate

$$\int_0^{\infty} \frac{\sin s \cos sx}{s}\, ds.$$

III. (a) State and prove the Modulation theorem for Fourier Sine and Cosine Transforms.

(b) Find Fourier transform of $f(t) = \begin{cases} 2\sin t, & 0 < t < a \\ 0, & t \geq a \end{cases}$

IV. Find the finite Fourier sine and cosine transform of $f(t) = t^2, 0 < t < l$.

SECTION-B (2x5.5=11)

V. (a) Solve

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-1}\sin t,\ y(0) = 0,\ y'(0) = 1.$$

(b) Solve $\frac{d^2x}{dt^2} + x = t\cos 2t,\ t > O$ where $x'(0) = 0$,

$x(0) = 0$.

VI. Solve $\frac{dx}{dt} - 2y = t,\ \frac{dy}{dt} - 4x + 2y = 0,$

when $x(0) = 3, y(0) = 0$.

VII. Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} + xt,$ where $u(x, 0) = 0, u, (x, 0) = 0$ and

$u(O, t) = 0$, where $u(x, t)$ is bounded.

VIII. Give the solution of Heat Conduction problem by Fourier Sine Transform.

SECTION-C
Compulsory Question

IX. Answer the following :

(a) State Fourier Integral formula.

(b) State Dirichlet's condition.

(c) Define the relation between Laplace and Fourier transform.

(d) Define the symmetric form of Fourier.

(e) State and prove change of scale property for complex Fourter transform.

(f) State Wave equation for second order partial differential equation.

(g) Solve $\frac{d^2y}{dt^2} + \frac{dy}{dt} = 2$, where $y(0) = 3, y'(0) = 1$.

(h) Find $y = \psi$ satisfying $\frac{d^2y}{dx^2} + \pi^2 y = 0$.

(i) Solve $(D^4 + 2D^2 + 1)x = 0$.

(j) Define the problem of heat conduction.

(1.4x10=14)