Total No. of Questions :8] **Total No. of Printed Pages :4]**

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B.A./B.Sc (General) Ist Semester (0001) Examination
0044
MATHEMATICS
Paper : II
(Calculus-I)

Time : 3 Hours] **[Max. Marks : 30**

Note :- Attempt five questions in all, selelcting at least two questions from each Section.

Section-A

1. (a) Solve for x the Inequality $\frac{x+2}{n-2} < \frac{4n-1}{2n-3}$.

   (b) Prove that $\left|x-\frac{1}{2}\right| < \frac{1}{3}$ iff $\frac{1}{11} < \frac{1-x}{1+x} < \frac{5}{7}$. 3,3

2. (a) State order completeness property of reals. Does the set of rational numbers posses this property ? Justify your answer.

(b) Find the least upper bound and greatest lower bound of the set $S = \left\{ \frac{2 - x}{1 - x};\ x > 0, x \neq 1 \right\}$. 3,3

3. (a) Is the union of two hounded sets a bounded set ? What do you say about its converse ? Justify your answer.

(b) If $f(x) = \left[x \frac{1}{x} \right]$, does $\underset{x \to 1/2}{\mathrm{Lt}}\ f(x)$ exist, explain your answer. 3,3

4. (a) Prove that if a function f(x) is continuous at a point a and f(a) = 0, then prove that these exists some neighbourhood of a where flx) possesses the same sign as that if f(a).

(b) Determine the values of a and b for which

$$\underset{n \to 0}{\mathrm{Lt}} \frac{x(1 + a\cosh x) - b\sinh x}{x^3} = 1$$ 3,3

Section-B

5. (a) By using Lagrange's mean value theorem prove that :

$$|\sin x - \sin y| \leq |x - y| \text{ for all } x, y \in R$$

(b) Calculate the approximate value of $\sqrt{24}$ to three decimal places by Taylor's expansion.

6. (a) Use Maclaurin'sv theorem to show that :

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{2} - \ldots\ldots\ldots\ldots$$

$$+ \frac{(-1)^{n-1}}{n} x^n - \frac{x^n}{(1+\theta x)^n}, 0 < \theta < 1.$$

(b) Use mean value theorem to show that :

$$\frac{\pi}{6} + \frac{2n-1}{\sqrt{3}} \leq \sin^{-1} x \leq \frac{\pi}{6} + \frac{2n-1}{2\sqrt{1-x^2}}$$

where $\frac{1}{2} \leq x < 1$. 3,3

7. (a) If $y = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \sinh^{-1} \frac{x}{a}$, show that :

$$\left(\frac{dy}{dx}\right)^2 = x^2 + a^2$$

(b) Prove that $\tanh^{-1} x = \frac{1}{2} \log \frac{1+x}{1-x}, -1 < x < 1$ and find its derivative also. 3,3

8. (a) Prove that :

$$\frac{d^n}{dx^n}\left(\frac{\log x}{x}\right) = \frac{(-1)^n \underline{\lfloor n}}{x^{n+1}}$$

$$\left[\log x - 1 - \frac{1}{2} - \frac{1}{3} \ldots\ldots\ldots\ldots, - \frac{1}{n}\right].$$

(b) If $y = \sin m (\sin\ x)$, show that :

$$(1 - x^2)\, y_{n+2} - (2n + 1)\, x y_{n+1} - (n^2 - m^2)\, y_n = 0$$

Hence show that:

$$y_n(0) = \begin{cases} 0, & \text{where n is even} \\ m(1^2 - m^2)(3^2 - m^2) \ldots\ldots\ldots L(n-2)^2 - m^2\,] & \end{cases}$$

when n is odd. 3,3