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Sub Code : 0341(1048) **Exam Code :** 0004

Exam : B.A./B.Sc. (General) 4th Semester

Subject : Mathematics

Paper : Paper-I Advanced Calculus-II

Time : 3 Hours ***Maximum Marks : 30***

Note : Attempt five questions in all, selecting at least two questions from each unit. All questions carry equal marks.

UNIT-I

1. (a) Prove that the sequence $\{a_n\}$ where

$$a_n = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \ldots + \frac{1}{2n}$$ is convergent. 3

(b) Prove that the sequence $\{a\ \}$ defined by $a_1 = \sqrt{2},\ a_n = \sqrt{2a_{n-1}}$ converges to 2. 3

2. (a) State and prove squeez principle.

(b) If $0 < s_1 < s_2$ and $s_n = \dfrac{2s_{n-1} + s_{n-2}}{s_{n-1} + s_{n-2}}$ show that $\{s_n\}$

converges to $\dfrac{3s_1 s_2}{2s_1 + s_2}$

3. (a) Show that $\lim_{n\to\infty} \dfrac{1}{n} \left(1 + 2^{1/2} + 3^{1/3} + \ldots\ldots\ldots + n^{1/n}\right) = 1$ 3

(b) Show that the sequence $\{a_n\}$, where

$a_n = 1 + \dfrac{1}{2} + \dfrac{1}{3} + \ldots\ldots\ldots\ldots + \dfrac{1}{n}$ does not

converge, by showing that it is not a Cauchy sequence. 3

4. (a) Show that f(x) = sin x is uniformly continous on $\left[0, \dfrac{\pi}{2}\right]$ 3

(b) Show that the function f defined by :

$$f(x) = \begin{cases} 1 & \text{if x is rational} \\ 0 & \text{if is irrational} \end{cases}$$

is discontinuous every where. 3

UNIT-II

5. (a) Discuss the convergence or divergence of

the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots\ldots\ldots$ 3

(b) Use Cauchy's condensation test to show that

that $\sum \frac{1}{np}$, $p > 0$ converges if $p > 1$ and

diverges if $p \leq 1$. 3

6. (a) Discuss the convergence or divergence of the

the series $\sum_{n=1}^{\infty} \frac{x^n}{3^n n^2}$ 3

(b) Discuss the convergence of the series

$\frac{x}{x+1} + \frac{x^2}{x+2} + \frac{x^3}{x+3} + \ldots\ldots\ldots, x > 0$ 3

7. (a) Examine the convergence or divergence of

the series $\frac{1^2}{2^2} + \frac{1^2.3^2}{2^2.4^2} x + \frac{1^2.3^2.5^2}{2^2.4^2.6^2} x^2 + \ldots\ldots, x>0.$ 3

(b) Show that the series $1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} +$

.... is convergent. 3

8. (a) Show that the series $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$ ....

is convergent for $-1 < x \leq 1$. 3

(b) Find the sum of the series

$$1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{2} + \frac{1}{7} + \frac{1}{9} + \frac{1}{11} + \frac{1}{4} + \dots\dots\dots .$$