Total No. of Questions :14] **Total No. of Printed Pages :4]**

(1126)

B.A./B.Sc (General) Vth Semester (0005) Examination

0443

MATHEMATICS

Paper : I

(Analysis-I)

Time : 3 Hours] **[Max. Marks : 30**

Note :- Attempt Five questions in all, selecting at least one question from each Section. All questions carry equal marks.

Section-A

1. (a) Prove that the set $\{.....2^{-3}, 2^{-2}, 2^{-1}, 1, 2, 2^{2}, 2^{3}, ...\}$ is countable.

(b) If $0 < a < b$, show that :

$$\left|\int_a^b \frac{\sin x}{x}\,dx\right| \leq \frac{2}{a}$$

2. (a) If f is continuum on [a, b], then $f \in R(x)$ on [a, b].

(b) Give an example of a bounded function f defined on a closed. interval such that |f| is R-integrable but f is not.

3. (a) If f is R-integrable on [a, b] and k is any real number, then k_f is also R-integrable and.

$$\int_a^b (k_f)(x)dx = k\int_a^b f(x)dx$$

(b) Show that :

$$\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+bx)^{m+n}}dx = \frac{1}{(a+b)^m\ a^n}\beta(m,n)$$

4. (a) Prove that :

$$\Gamma(m)\Gamma\left(m+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$$

(b) Show that :

$$\int_0^\infty \frac{x}{1+x^6}\ dx = \frac{\pi}{3\sqrt{3}}$$

Section-B

5. (a) Discuss the convergence of $\int_0^\infty \frac{x \tan^{-1} x}{(1+x^4)^{1/3}}\, dx$.

(b) Discuss the convergence of $\int_a^b \frac{dx}{(x-a)\sqrt{b-x}}$.

6. (a) If $\phi(x)$ is bounded and monotonic in $[a, \infty]$ and $\int_a^\infty f(x)dx$ is convergent at ∞, then prove that $\int_a^\infty f(x)\phi(x)dx$ is convergent at ∞.

(b) Show that $\int_0^\infty \frac{\sin^2 x}{x^2}\, dx$ convergent. Also using $\int_0^\infty \frac{\sin x}{x}\, dx = \frac{\pi}{2}$, show that $\int_0^\infty \frac{\sin^2 x}{x^2}\, dx = \frac{\pi}{2}$

7. (a) Show that $\int_0^{\pi/2} \sin x . \log \sin x \, dx$ is convergent and its value is $\log \frac{2}{e}$.

(b) Show that $\int_0^{\infty} \frac{\sin ax \sin bx}{x} \, dx$ converges to $\frac{1}{2} \log\left(\frac{a+b}{a-b}\right)$ where a + b are positve reals.

8. By applying rule of differentiation under integral sign, prove the following :

(a) $\int_0^{\pi/2} \log\left(\frac{a + b\sin\theta}{a - b\sin\theta}\right) \operatorname{cosec}\theta \, d\theta = \pi \sin^{-1} \frac{b}{a}.$

(b) $\int_0^{\pi/2} \frac{\log(1 + y\sin^2 x)}{\sin^2 x} \, dx = \pi \, [\sqrt{1+y} - 1].$