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B.A./B.Sc.(General) 6th Semester Examination

1047

MATHEMATICS

Paper : I (Analysis-II)

Time : 3 Hours] **[Max. Marks : 30**

Note :- Attempt five questions in all, selecting at least two from each Unit. All questions can equal marks.

Unit-I

1. (a) Let $A = \{(x,y) : -1 \le x \le 1, -1 \le y \le 1\}$
And $f : A \to R$ be defined by :

$$f(x,y)=\begin{cases} y & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ ix irrational} \end{cases}$$

Show that $\int_{-1}^{1}\left(\int_{-1}^{1} f\,(x, y)dy\right)dx$ exists and the other repeated integral is not defined.

(b) Change the order of integration and hence evaluate

evaluate $\int_0^a \int_x^{\frac{a^2}{x}} (x + y)\, dx\, dy.$

2. (a) Find the area of the region bounded between the parabolas $y^2 = 4\,ax$ and $x^2 = 4\,ay$, where $a > 0$.

(b) Evaluate $\iiint x\,y\,z(x^2 + y^2 + z^2)\, dx\, dy\, dz$ over

$x^2 + y^2 + z^2 = a$ in positive octant.

3. (a) Show that $\iiint (x + y + z)^9\, dx\, dy\, dz$ over the

regoin defined by $x \geq 0,\ y \geq 0,\ z \geq 0,$

$x + y + z = 1$ is $\dfrac{1}{24}$.

(b) Show that $\vec{F} = (2\,x\,y + z^3)\,\hat{i} + x^2\hat{j} + 3\,x\,z^2\hat{k}$ is a conservative force field. Find the scalar potential. Also find the work done in movingan object in this field from (1, -2, 1) to (3, 1, 4).

4. (a) State and prove Gauss's divergence theorem.

(b) Verify Stokes' theorem for the vector point function $\vec{F} = z\,\hat{i} + x\,\hat{j} + y\,\hat{k}$, where curve is the unit circle in the XY plane bounding the semi-sphere $z=\sqrt{1 - x^2 - y^2}$.

Unit-II

5. (a) Show that sequence $\{f_n(x)\}$ where $f_n(x) = \dfrac{n}{x+n}$ is uniformly convergent on [0, k], where k is any positive real number but is not uniformly convergent on $[0,\infty]$.

   (b) Show that the series $\sum_{n=1}^{\infty} \dfrac{\cos nx}{n}$ converges uniformly in $(0, 2\pi)$.

6. (a) Test for uniform convergence and term by term integration of the series:

$$\sum_{n=1}^{\infty} \frac{x}{(n+x^2)^2}, 0 \le x \le 1$$

   (b) Show that the series $\sum_{n=1}^{\infty} \dfrac{1}{n^2 + n^4 x^2}$ is uniformly convergent fol all x and.it can be differentiated term by term.

7. (a) Prove that the series $x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \ldots$ converges for $-1 < x \le 1$.

(b) Prove that :

$$\sin^{-1} x = x + \sum_{n=1}^{\infty} \frac{1.3.5....(2n-1)}{2.4.6.....2n}$$

$$\frac{x^{2n+1}}{2n+1} \; \forall \; x \in [-1, l].$$

Hence deduce that $\frac{\pi}{2} = 1 + \frac{1}{2} \cdot \frac{1}{3} + \frac{1.3}{2.4} \cdot \frac{1}{5} + ...$

2 8. (a) Find a series of sines and cosines of multiples of x which represents $x + x$ in $(-\pi, \pi)$, Hence show

that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^3} + ...$

(b) If $f(x) = \begin{cases} \frac{\pi}{3}; & 0 \leq x \leq \frac{\pi}{3} \\ 0; & \frac{\pi}{3} \leq x \leq \frac{2\pi}{3} \\ \frac{-\pi}{3}; & \frac{2\pi}{3} \leq x \leq \pi \end{cases}$

then show that:

$$f(x) = \frac{2}{\sqrt{3}} \left[\cos x - \frac{\cos 5x}{5} + \frac{\cos 7x}{7} - .... \right].$$

Hence deduce that $\frac{\pi}{2\sqrt{3}} = 1 - \frac{1}{5} + \frac{1}{7} - \frac{1}{11} + ...$