Roll No. .... **Total Pages : 3**

8656/MH

AS-2057
PARTIAL DIFFERENTIAL EQUATION - V
(Semester-II)

Time Allowed : 3 Hours] **[Maximum Marks : 36**

Note :- Attempt two questions each from Section A and B carrying 5.5 marks each, and the entire Section C consisting of 7 short answer type questions carrying 2 marks each.

SECTION-A

I. (a) Construct a PDE by eliminating a, b and c from

$$z = a(x + y) + b(x - y) + abt + c.$$

(b) Form a PDE by eliminating arbitrary functions from

$$z = f(x^2 - y) + g(x^2 + y).$$

II. Find the complete integral of $x^2 p^2 + y^2 q^2 = z^2$.

III. Solve the PDE :

$$p \tan x + q \tan y = \tan z.$$

IV. Apply Charpit's method to find complete integral of

$$z^2(p^2z^2+q^2)=1.$$

SECTION-B

V. Solve the PDE:

$$\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial y^2} = \frac{4x}{y^2} - \frac{y}{x^2}$$

VI. Solve :

$$\frac{1}{x^2}\frac{\partial^2 z}{\partial x^2} - \frac{1}{x^3}\frac{\partial z}{\partial x} \quad \frac{1}{y^2}\frac{\partial^2 z}{\partial x^2} - \frac{1}{y^3}\frac{\partial z}{\partial y}$$

VII. Solve the one-dimensional heat equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{m}\frac{\partial u}{\partial t}$

in $0 \le x \le \pi$, $t > 0$ given u remains finite as $t \to \infty$.

VIII. Find the general solution of one-dimensional wave

equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ and find particular solution

for which $u = f(x)$, $\frac{\partial u}{\partial t} = g(x)$ at $t = 0$.

SECTION-C

IX. Attempt all the following :

(a) Solve : $z = px + qy + c\sqrt{1+p^2+q^2}$.

(b) Solve the PDE : $xp - yq = \frac{y^2 - x^2}{z}$

(c) Solve the PDE : $yzp + zxq = xy$.

(d) Solve the DPE :

$$(D^3 - 4D^2D' + 4DD'^2)z = 4 \sin(2x + y).$$

(e) Find the surface satisfying $(D^2 - 2DD' + D'^2)z = 0$ and conditions that $bz = y^2$ when $x = 0$ and $az = x^2$ when $y = 0$.

(f) Explain the D' Alembert's procedure of finding the solution of $\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2}$.

(g) Solve $z = px + qy + p^2 + q^2$ using Charpit's method.