Roll No. .... Total Pages : 6

8675/MH

AS-2057

ANALYSIS-II

Paper-V

(Semester-IV)

Time Allowed : 3 Hours] [Maximum Marks : 36

Note :- The candidates are required to attempt two questions each from Sections A and B carrying 5.5 marks each and the entire Section C consisting of 8 short answer type questions carrying 14 marks.

SECTION-A

1. (a) Show that $\lim_{n\to\infty} a_n = 0$, where $\{a_n\}$ is a sequence

with $\lim_{n\to\infty} \frac{a_n + 1}{a_n} = 1, |l| < 1.$ 3

(b) Show that :

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\dots+\frac{1}{\sqrt{n^2+1}}\right]=1.$$

2.5

2. (a) Show that for any real number

$$x,\ \lim_{n\to\infty}\frac{x^n}{n!}=0.$$ 3

(b) Test the convergence of the series

$$\sum \frac{(n^3+1)^{1/3}-n}{\log n}$$ 2.5

3. (a) Show that the series

$$\frac{1}{(\log 2)^P}+\frac{1}{(\log 3)^P}+\dots\frac{1}{(\log n)^P}+\dots\dots$$

diverges for $p > 0$. 3

(b) Test the convergence of

$$\sum_{n=1}^{\infty} C^{-n^2}.$$ 2.5

4. (a) Establish the divergence of the series

$$2-\frac{3}{2}+\frac{4}{3}-\frac{5}{3}+\dots\dots$$ 2.5

(b) Show that the series $\sum \frac{(-1)^{n+1}}{n^P}$ converges conditionally for $0 < p \leq 1$ and converges absolutely for $p > 1$. 3

SECTION-B

5. Prove that the series $\sum (-1)^n \frac{x^2 + n}{n^2}$ converges uniformly in every bounded interval but does not converge absolutely for any real value of x. 5.5

6. Prove that the series $\sum \frac{\sin n\theta}{n^P}$ converges uniformly for all $p > 0$ in $[\alpha, 2\pi - \alpha]$ where $0 < \alpha < \pi$. 5.5

7. Show that the series :

$$\frac{x}{1+x} + \frac{x}{(1+x)(1+2x)} + \frac{x}{(1+2x)(1+3x)} + ...$$

is uniformly convergent on [a, b], $a > 0$ but only pointwise in [0, b]. 5.5

8. Prove that if a power series $\sum a_n x^n$ diverge for some $x = x_0$, then it diverges for every x, for which

$|x| > |x_0|$. 5.5

SECTION-C

9. (a) State Abel's theorems on power series. 1

(b) Determine the radius of convergence and the exact interval of convergence of the series.

$$\sum_{n=1}^{\infty} \frac{x^n}{n!}$$ 2

(c) What is the difference between the concept of pointwise and uniform convergence of a sequence of function. Also give example of each type of sequence. 2

(d) Give an. example of a function to show that limit of differentials is not equal to the differential of the limit. 2

(e) Show that for any fixed value of x the series

$\sum \frac{\sin nx}{n^2}$ is convergent. 2

(f) Differentiate between the terms absolute and conditional convergence. 2

(g) Test the convergence of the series $\sum \frac{1}{n^{1+1/n}}$ 2

(h) State Abel's and Dirichlet's tests. 1