

8697/MH

AS-2057
DISCRETE MATHEMATICS-II
Paper-IV
Semester-VI

Time Allowed : 3 Hours]**[Maximum Marks : 36**

Note :- The candidates are required to attempt two questions each from Sections A and B carrying marks 5.5 each and the entire Section C consisting of 10 questions carrying 1.4 marks each.

SECTION-A

1. (a) Give Big-O estimate for $f(n) = 3n \log n! + (n^2 + 3) \log n$.
(b) Prove that $f(x) = 8x^3 + 5x^2 + 7$ is $\Omega(g(x))$, where $g(x) = x^3$.
2. (a) Solve $S_n + 5S_{n-1} + 6S_{n-2} = 3n^2 - 2n + 1$.
(b) Find sequence whose generating function is $\frac{1}{1 - z - z^2}$.
3. (a) Solve recurrence relation $S(n) - 4S(n - 2) = 0$, $S(0) = 10$, $S(1) = 1$ for $n \geq 0$ using generating function.
(b) For $S(n) = 3^n$ verify that $G(S * a, Z) = G(S, Z) G(a, Z)$.

4. (a) For the recurrence relation $a_n = 8a_{n-1} + 10^{n-1}$ with initial condition $a_0 = 1$. Find the generating function and the explicit formula for a_n .
- (b) Find generating function for the sequence of Fibonacci numbers.

SECTION-B

5. (a) Prove that set D_n of all positive divisors of n is a bounded distributive lattice.
- (b) Prove that for a bounded distributive lattice L , the complements are unique if they exist.
6. (a) Find the circuit $(x_1 - ((x_2 \cdot \overline{x_3}) + (\overline{x_2} \cdot x_3))) + (\overline{x_1} \cdot x_2 \cdot x_3)$.
- (b) Simplify the Boolean expression :

$$F(X, Y, Z) = (\overline{X} \cdot Z) + (V \cdot Z) + (V \cdot \overline{D})$$
 and write in min. term normal form.
7. (a) Minimize the logic programme using K map:

$$f(A, B, C, D) = \sum (0, 1, 2, 3, 5, 7, 8, 9, 10, 4).$$
- (b) Reduce using Boolean rules $xy + xz + yz = xy + (x \oplus y)z$.
8. (a) Write the function $x \vee y'$ in the disjunction normal form in three variables x, y and z .
- (b) Simplify the Boolean expression and make circuit diagram using NAND gate only.

$$F(A, B, C, D) = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}C\overline{D} + \overline{A}B\overline{C}\overline{D} + \overline{A}B\overline{C}D + \overline{A}B\overline{C}\overline{D} + \overline{A}B\overline{C}D + A\overline{B}\overline{C}\overline{D} + A\overline{B}\overline{C}D$$

SECTION-C

9. (a) Define ceiling function and characteristic function.
(b) If f be mod-11 function then find the value of $f(-253)$.
(c) The numeric value of a defined as $a_r = \begin{cases} 2, 0 \leq r \leq 3 \\ 2^{-r} + 5, r \geq 4 \end{cases}$, find $S^{-2} a$.
(d) Determine $C(5, 3)$ by recursive definition of binomial coefficient.
(e) Write short note on recursion.
(f) Show that n , n th root of unity forms a group under multiplication.
(g) Prove that inverse of an element of group is unique.
(h) Draw operation table of $G = \{0, 1, 2, 3, 4, 5\}$ under multiplication modulo 6.
(i) Define ring and sub ring.
(j) Prove that dual of distribution lattice is distributive.

10x1.4=14

